Solving equations numerically : Rearrange to form iterative equations

Maths

Mr Clasper

Solving equations numerically : Rearrange to form iterative equations

1. Show that the equation $x^2 + 2x = 3$ can be rearranged to each of the following equations.

a) $x = \sqrt{3 - 2x}$

b) $x = \dfrac{3 - x^2}{2}$

2. Show that the following iterative formulae can be rearranged from the equation $x^2 - 5x + 9 = 0$

a) $x_{n+1} = \sqrt{5x_n - 9}$

b) $x_{n+1} = 5 - \dfrac{9}{x_n}$

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3. Show that the equation $x^3 + 2x = 1$ can be rearranged to each of the following equations.

a) $x = \sqrt[3]{1 - 2x}$

b) $x = \sqrt{\dfrac{1}{x} - 2}$

c) $x = \dfrac{1 - 2x}{x^2}$

4. Millie is solving the following equation.

$$x^3 + x^2 = 5$$

She wants to form an iterative equation to use.

Show that one possible iterative equation is

$$x_{n+1} = \sqrt{\dfrac{5}{x_n} - x_n}$$

Answers

Solving equations numerically : Rearrange to form iterative equations

1. Show that the equation $x^2 + 2x = 3$ can be rearranged to each of the following equations.

a) $x = \sqrt{3 - 2x}$

$x^2 = 3 - 2x$

$x = \sqrt{3 - 2x}$

b) $x = \dfrac{3 - x^2}{2}$

$2x = 3 - x^2$

$x = \dfrac{3 - x^2}{2}$

2. Show that the following iterative formulae can be rearranged from the equation $x^2 - 5x + 9 = 0$

a) $x_{n+1} = \sqrt{5x_n - 9}$

$x^2 = 5x - 9$

$x = \sqrt{5x - 9}$

b) $x_{n+1} = 5 - \dfrac{9}{x_n}$

$x - 5 + \dfrac{9}{x} = 0$

$x = 5 - \dfrac{9}{x}$

Solving equations numerically : Rearrange to form iterative equations

3. Show that the equation $x^3 + 2x = 1$ can be rearranged to each of the following equations.

a) $x = \sqrt[3]{1 - 2x}$

    First step

    $x^3 = 1 - 2x$

b) $x = \sqrt{\dfrac{1}{x} - 2}$

    First step

    $x^2 + 2 = \dfrac{1}{x}$

c) $x = \dfrac{1 - 2x}{x^2}$

    First step

    $x^3 = 1 - 2x$

4. Millie is solving the following equation.

$$x^3 + x^2 = 5$$

She wants to form an iterative equation to use.

Show that one possible iterative equation is

$$x_{n+1} = \sqrt{\dfrac{5}{x_n} - x_n}$$

$x^3 + x^2 = 5$

$x^2 + x = \dfrac{5}{x}$

$x^2 = \dfrac{5}{x} - x$

$x = \sqrt{\dfrac{5}{x} - x}$