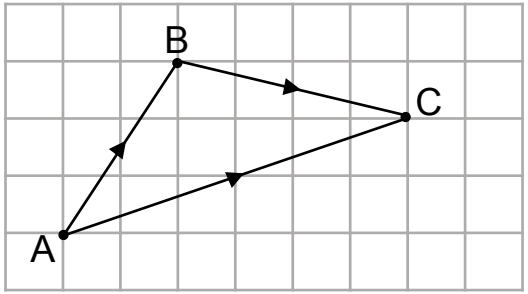
Add and subtract two column vectors to give a resultant vector

Miss Davies
Maths

Add and subtract two column vectors

1. Look at the diagram.



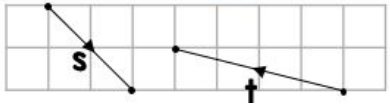
(a) Write the following as column vectors

(i) $\overrightarrow{AB}$ (ii) $\overrightarrow{BC}$ (iii) $\overrightarrow{AC}$

(b) Use the column vectors from part (a) to show that $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

(c) Now use your answers to part (a) to show that $\overrightarrow{AC} - \overrightarrow{BC} = \overrightarrow{AB}$

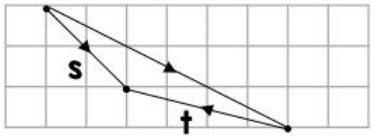
2. $s = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$ $t = \begin{pmatrix} -4 \\ 1 \end{pmatrix}$

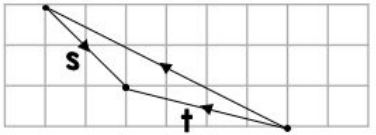


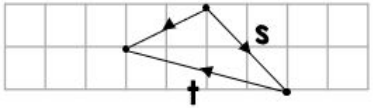
(a) Calculate the following

(i) $s + t$ (ii) $s - t$ (iii) $t - s$

(b) Match each of the resultant vectors from part (a) to the following diagrams.







Add and subtract two column vectors

3. If $\mathbf{q} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$ $\mathbf{r} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$ $\mathbf{s} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$

Find:

(a) $\mathbf{q} + \mathbf{r}$ (b) $\mathbf{q} + \mathbf{s}$ (c) $\mathbf{r} + \mathbf{s}$

(d) $\mathbf{q} - \mathbf{r}$ (e) $\mathbf{q} - \mathbf{s}$ (f) $\mathbf{r} - \mathbf{s}$

(g) $\mathbf{r} - \mathbf{q}$ (h) $\mathbf{s} - \mathbf{q}$ (i) $\mathbf{s} - \mathbf{r}$

4. If $\mathbf{x} = \begin{pmatrix} -3 \\ 0 \end{pmatrix}$, $\mathbf{y} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ and $\mathbf{z} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$

A calculation involving x and y gives the following resultant vectors.

What could the calculation be?

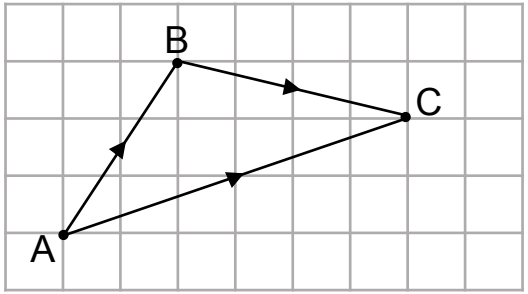
(a) $\begin{pmatrix} 1 \\ -3 \end{pmatrix}$ (b) $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$

(c) $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$ (d) $\begin{pmatrix} -1 \\ -5 \end{pmatrix}$

Answers

Add and subtract two column vectors

1. Look at the diagram.



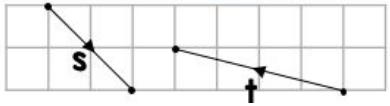
(a) Write the following as column vectors

(i) $\overrightarrow{AB}$ $\begin{pmatrix}2\\3\end{pmatrix}$ (ii) $\overrightarrow{BC}$ $\begin{pmatrix}4\\-1\end{pmatrix}$ (iii) $\overrightarrow{AC}$ $\begin{pmatrix}6\\2\end{pmatrix}$

(b) Use the column vectors from part (a) to show that $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$ $\begin{pmatrix}2+4\\3-1\end{pmatrix} = \begin{pmatrix}6\\2\end{pmatrix}$

(c) Now use your answers to part (a) to show that $\overrightarrow{AC} - \overrightarrow{BC} = \overrightarrow{AB}$ $\begin{pmatrix}6-4\\2--1\end{pmatrix} = \begin{pmatrix}2\\3\end{pmatrix}$

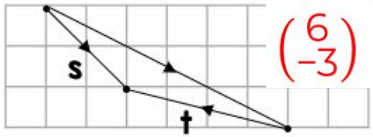
2. $s = \begin{pmatrix}2\\-2\end{pmatrix}$ $t = \begin{pmatrix}-4\\1\end{pmatrix}$



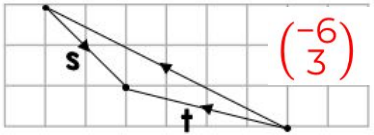
(a) Calculate the following

(i) $s + t$ $\begin{pmatrix}-2\\-1\end{pmatrix}$ (ii) $s - t$ $\begin{pmatrix}6\\-3\end{pmatrix}$ (iii) $t - s$ $\begin{pmatrix}-6\\3\end{pmatrix}$

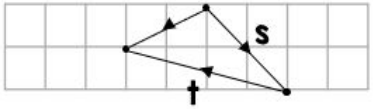
(b) Match each of the resultant vectors from part (a) to the following diagrams.



$\begin{pmatrix}6\\-3\end{pmatrix}$



$\begin{pmatrix}-6\\3\end{pmatrix}$



$\begin{pmatrix}-2\\-1\end{pmatrix}$

Add and subtract two column vectors

3. If $\mathbf{q} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$ $\mathbf{r} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$ $\mathbf{s} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$

Find:

(a) $\mathbf{q} + \mathbf{r}$ $\begin{pmatrix} 9 \\ 3 \end{pmatrix}$

(b) $\mathbf{q} + \mathbf{s}$ $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$

(c) $\mathbf{r} + \mathbf{s}$ $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$

(d) $\mathbf{q} - \mathbf{r}$ $\begin{pmatrix} -1 \\ 7 \end{pmatrix}$

(e) $\mathbf{q} - \mathbf{s}$ $\begin{pmatrix} 7 \\ 7 \end{pmatrix}$

(f) $\mathbf{r} - \mathbf{s}$ $\begin{pmatrix} 8 \\ 0 \end{pmatrix}$

(g) $\mathbf{r} - \mathbf{q}$ $\begin{pmatrix} 1 \\ -7 \end{pmatrix}$

(h) $\mathbf{s} - \mathbf{q}$ $\begin{pmatrix} -7 \\ -7 \end{pmatrix}$

(i) $\mathbf{s} - \mathbf{r}$ $\begin{pmatrix} -8 \\ 0 \end{pmatrix}$

4. If $x = \begin{pmatrix} -3 \\ 0 \end{pmatrix}$, $y = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ and $z = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$

A calculation involving x and y gives the following resultant vectors.

What could the calculation be?

(a) $\begin{pmatrix} -1 \\ -3 \end{pmatrix}$ x + y or y + x

(b) $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$ z - y

(c) $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$ y - z

(d) $\begin{pmatrix} -1 \\ -5 \end{pmatrix}$ x + z